Increased Precision of Zernike Fit to Noisy Truncated Phase Maps

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Abstract: When a polynomial set fails to meet the conditions of orthogonality, errors appear in the calculated coefficients. Restoring orthogonality by fitting a custom Zernike set to noisy phase maps provides increased precision in low-order coefficients.
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1. Background

Zernike polynomials have long been an industry standard for specifying and analyzing the wavefront of an optical system. The 37-term Fringe Zernike polynomial set, commonly used to describe the residual optical aberrations in a system under test, is orthogonal when the pupil is circular and continuous [1]. When real measurements are made, the circular pupil is sampled and becomes non-continuous. A discretely sampled pupil fails to meet one of the conditions for orthogonality. When many points are sampled across the pupil, the phase maps can be treated as being continuous and the Zernike set is approximately orthogonal. Resulting coefficients are calculated with high precision across a number of image iterations, even in the presence of camera noise. Dense sampling is often seen in an interferometer and camera setup.

Another common test setup comprises a Shack-Hartmann wavefront sensor with associated software to reconstruct the phase map from the spot array information. The wavefront is sparsely sampled and oftentimes Zernike coefficients are fit to determine the aberration content of the pupil. This setup works well for circular pupils, even when the sensor contains fewer than 50 discrete points across the aperture and fails to meet one orthogonality condition. When the aperture is truncated, however, a second condition for orthogonality is no longer met and significant errors can arise in the Zernike fit[2]. Engineers using a Shack-Hartmann setup as live feedback during system alignment have difficulty correlating these Zernike coefficients with physical parameters. We examine restoring orthogonality to a truncated system using a modified set of Zernike polynomials, designed to be orthogonal over the geometry present in our system. We refer to this set as "Truncated Zernikes" [3]. We demonstrate greatly increased stability of these Truncated Zernike coefficients over a series of noisy phase maps. The ways in which these truncated coefficients manifest in a physical system is not well-understood; consequently, the test engineer has difficulty aligning a system without using an image processing and optimization algorithm. Implementing Truncated Zernikes to system analysis during the design and tolerancing phases of a project may provide valuable insight and assist with the alignment process.

2. Simulation

We simulate a Shack-Hartmann phase map of a circular aperture with fringe Zernike coefficients Z5 = 1, Z7 = 1, Z9 = 1 and Z16 = 1. All other coefficients are zero and the aperture is 38 pixels in diameter. We then truncate the aperture in one dimension to create an asymmetric pupil, and generate 30 noisy maps using Gaussian noise consistent with a typical Shack-Hartmann test setup. Each datamap is fit with the first 25 fringe Zernike terms to determine precision of the fit across noisy images. Since the simulated Shack-Hartman test setup samples the pupil at only 38 points across the pupil, fitting the phase maps with fringe Zernikes fails to meet one condition of orthogonality. Furthermore, the aperture is truncated in one dimension and is no longer a unit circle; consequently, a second condition of orthogonality is unmet leading to large errors in the calculation of low-order Zernike coefficients. Figure 1 shows the non-zero Zernike term coefficients fit to each noisy truncated pupil. The symmetric terms, Z9 and Z16, display a small variation with noise but remain relatively close to the noiseless coeffient of one. The asymmetric terms, however, display a large variation when noise is added to the pupil; the Z5 and Z7 terms vary by up to 40 percent of the noiseless coefficient.

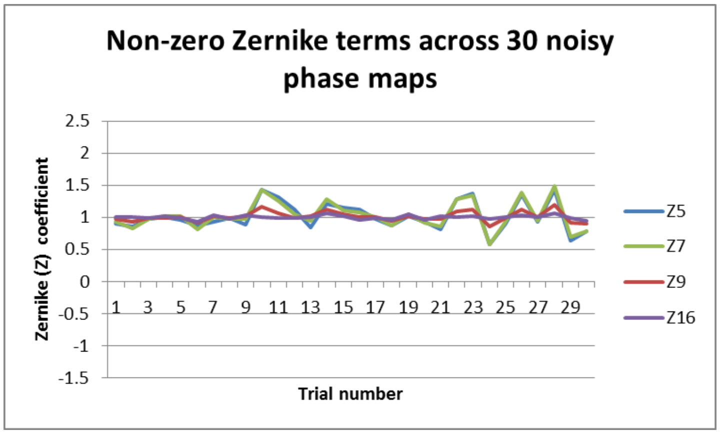

Figure 1 Fringe Zernike terms across 30 noise iterations of truncated pupil

One might consider decreasing the order of the Zernike fit so that only the first 9 or 16 terms are included. Figure 2 shows the effects of this strategy on Z7. Fitting a smaller number of terms does indeed increase stability of the coefficient; noise has a decreased impact when fitting 16 terms and even moreso with a 9 term fit. However, a close examination reveals that the 9 term fit also includes a bias over the noiseless Z7 term. A test engineer aligning a system based upon data from a 9-term fit may be unaware of this bias and could induce errors into the lens attempting to drive this term to zero.

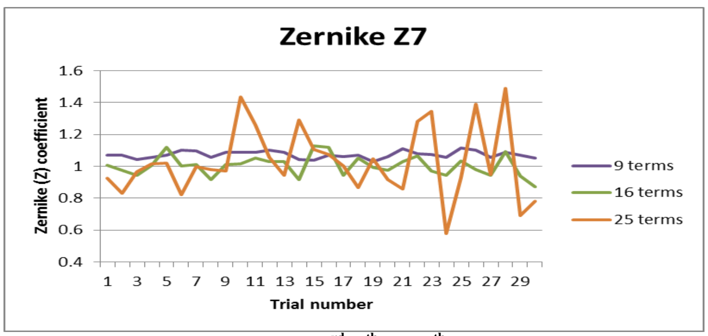

Figure 2 Fringe Zernike term Z7 from 3rd, 5th and 7th order fits to noisy truncated phase map

Instead of Fringe Zernikes, we can fit the pupil with an orthogonal polynomial set tailored to the irregular geometry of our system. We used a Truncated Zernike set very similar to the Fringe Zernikes, but orthogonal over the truncated aperture. When one additional orthogonality condition is met using the Truncated Zernike set, the polynomial fit becomes much more stable across different noise iterations, as seen in Figure 3. It is important to note that terms do not directly correlate between the two sets. Comparing Figures 1 and 3, one may note that terms Z9 and Z16 maintain an average value of 1 across the pupil iterations, while the truncated terms TZ9 and TZ16 have average values close to 2. Additional Truncated terms TZ4, TZ10, TZ12 and TZ14 have non-zero contributions, while their corresponding Fringe Zernike terms were all nominally zero.

3. Conclusions

As the amount of truncation in the pupil increases, Fringe Zernike coefficients become more unstable with noise iterations, making the Truncated Zernike set more valuable. Furthermore, Truncated Zernikes provide a more accurate representation of the data that is actually useful, while ignoring the eliminated portion of the circular pupil. Traditionally, Fringe Zernikes have been a useful tool to the test engineer because their low order terms match closely with the Seidel aberrations, commonly used in lens design and optimization. However, using Fringe Zernikes in a truncated system can be misleading to the alignment engineer: a pupil with a very small wavefront RMS value may have aberrations that are balanced in the used portion of the pupil, while contributions in the occluded region combine to artificially high values. These Zernike fits are often unstable with small changes to the wavefront (*i.e.* noise) and trusting Fringe Zernikes in these cases can lead to sub-optimal alignment. We have shown that replacing the traditional Fringe Zernike set with a Truncated Zernike set, orthogonal over the geometry of the pupil, greatly increases stability of the coefficients across noisy phase maps.

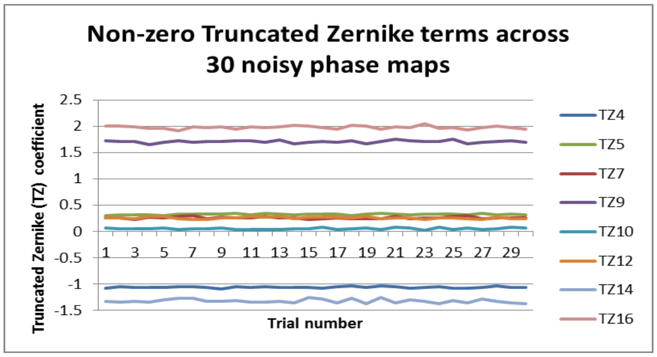

Figure 3 Truncated Zernike terms across 30 noise iterations of truncated pupil

The challenge we are faced with using Truncated Zernikes arrives when we attempt to use them as system alignment tools with real-time feedback. The Fringe Zernike set used to create our pupil simulation contains only four non-zero terms, three of which are simple to correct with physical system parameters. Z9 and Z16 are third- and fifth-order spherical terms and can generally be reduced by an adjustment of an airspace within the lens. Axial Z7 is a coma term and can be reduced by decentering a lens element in X and Y. When fit with Truncated Zernikes, as seen in Figure 3, the TZ terms do not have a one-to-one correlation with fringe Zernikes, nor with the Seidel aberrations. The same pupil fit with truncated Zernikes contains eight different non-zero terms. The ways in which these truncated coefficients manifest in a physical system is not intuitive, necessitating a custom real-time software interface capable of displaying measured Truncated Zernikes compared to nominal Truncated Zernikes for the system under test.

4. References

1. Shannon, R.R., *The Art and Science of Optical Design*. 1997: Cambridge University Press.
2. Genberg, V.L., G.J. Michels, and K.B. Doyle. *Orthogonality of Zernike polynomials*. 2002.
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