

# Analytical Approach to Evaluate Maximum Gravitational Sag and its Variations of Glass Substrate for Flat Panel Displays

## Technical Information Paper

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### TIP 307

Issued: April 2021

Supersedes: November 2004

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### Abstract

We present analytical formulas to evaluate the gravitational sag of glass substrate for Flat Panel Displays (FPD). The formulas are derived for the parallel line supports (knife-edges). Past studies and comparison with finite element analysis in this paper show that a continuous line support is a very close approximation to collinear point supports with the kind of support spacing typical in today's cassette designs. Two, three and four supporting lines are considered. A new concept of initial shapes of the glass substrate is introduced to enable evaluation of sag variations caused by various process factors analytically and numerically. We show that both the sag magnitude of the glass substrate and its variations due to various process factors may be drastically decreased with increased number of horizontal supports. The analytical results are verified numerically with the finite-element method.

### 1. Introduction

There is an increasing need to run larger glass substrate by

FPD panel manufacturers. The rapid growth of the substrate size contributes greatly to the low cost manufacturing of FPD. However, it also poses big challenges to FPD production processes. One of those challenges is the enlarged gravitational sag of the glass substrate. The gravitational sag is defined as the deviation from a flat plane that occurs when a sheet of glass is supported horizontally and allowed to naturally bend due to its own weight. The magnitude of the gravitational sag is a function of the location within a sheet and it has zero value at locations where a glass substrate is physically supported. As one moves away from a supporting location, sagging of the glass substrate increases and eventually reaches its maximum value. If the edges of a glass substrate are not supported, sagging of glass substrate also takes place at the edges. Therefore, the gravitational sag can take its maximum either at these edges of the substrate or somewhere between supporting lines. For practical

use, the most important number of the gravitational sag is its maximum value. Therefore, we use the term “sag” to denote the maximum gravitational sag in what follows.

As the substrate size grows, panel makers and equipment vendors need to carefully design substrate handling systems to avoid glass breakage or scratches during processing. Glass breakage may occur at various process steps if glass handling systems are not properly designed and the sag magnitude of the glass substrate becomes too large. Scratches or breakage may also occur in glass cassettes due to sag variations. Major causes for sag variations include variations of the unsupported spans of the glass sheet between cassette's support pins, thickness variations of the glass substrate and variations and non-uniformity of FPD manufacturing processes such as film deposition. Sag variations are unavoidable to some extent because none of these causes can be eliminated completely.

In previous studies [1-3], the sag magnitude has been calculated as a function of the number of supporting lines and the optimum support positions (where the sag is minimum) have been identified. In this paper, we extend the approach proposed in [1-3] towards some non-flat initial shapes of the glass substrate and perform the sensitivity study for the large size (Generation 7) glass substrates. We introduced the non-flat initial shape to the glass substrate as a proxy for sag variations caused by previously described factors. To model sag variations due to various factors such as unsupported length of cassette, glass substrate thickness, or film stress, we introduced convex or concave initial shape to the glass substrate. This study is aimed to show how given sag variations (whatever the causes are) can be reduced by increasing number of supporting lines.

The remainder of the paper is organized as follows. In Section 2, we briefly describe the analytical approach to calculate the FPD glass substrate sag, present formulas for calculation of the substrate shape and validate the approach with numerical calculations performed using the finite element method (FEM). In Section 3, we discuss the sag sensitivity to the optimum support position and the effect of the initial shape of the glass substrate. Main conclusions are drawn in Section 4.

## 2. Analysis

To determine the shape of the glass substrate resting on knife-edge supports, we use the same approach as for the analysis of the beam deflection [4]. Similarly, the deflection of the glass substrate on vertical direction  $w(x)$  with the homogeneous flexural rigidity can be found from the fourth-order ordinary differential equation [4]:

$$\frac{EI}{(1-\nu^2)} \frac{d^4 w}{dx^4} = -P_0 \quad (1)$$

where  $x$  is the transverse coordinate (see Figure 1),

$w$  is the deflection of the sheet in the vertical direction (see Figure 2),  $E$  is the Young modulus,  $\nu$  is the Poisson's ratio,  $I$  is the moment of inertia of the cross-sectional area and  $P_0$  is the weight per unit of length.

The factor  $(1-\nu^2)$  in equation (1), applies for plates though not for beams (for details, see [5]). Substitution of values of  $I$  and  $P_0$  and straightforward integration of

(1) results in the polynomial equation

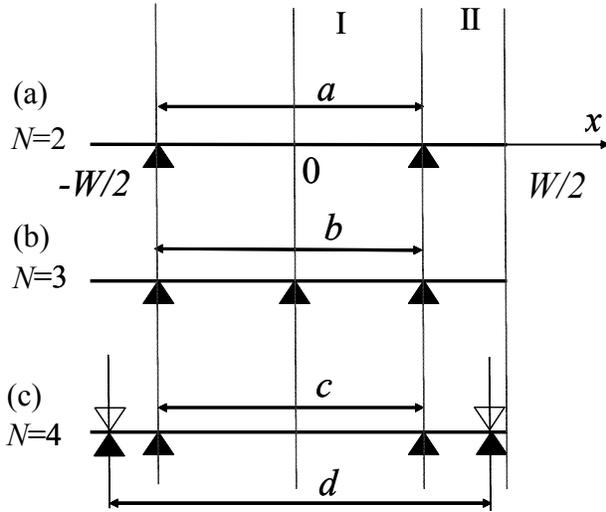
$$w(x) = R(-x^4 + Q_3 x^3 + Q_2 x^2 + Q_1 x + Q_0) \quad (2)$$

where

$$R = \frac{\rho g (1-\nu^2)}{2Et^2} \quad (3)$$

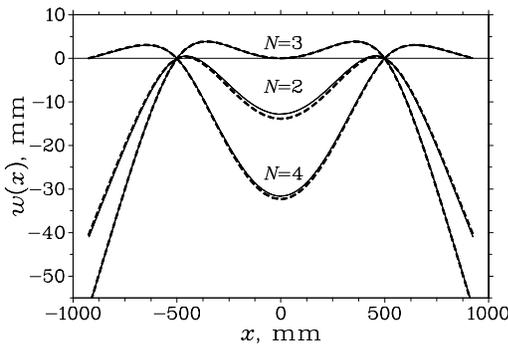
and  $\rho$  is the glass density,  $g = 9.8 \text{ m/s}^2$  is the acceleration of gravity and  $t$  is the substrate thickness. The coefficients  $Q_i$  ( $i = 0,1,2,3$ ) of the polynomial in (2) are to be found from boundary conditions for each section of the plate (denoted by I and II in Figure 1). Boundary conditions include symmetry considerations [  $w(x) = w(-x)$  ], continuity at the points of support and fixed support and free end conditions [4]. As a result, four algebraic equations for four unknown coefficients  $Q_i$  can be written for the sections I and II. The straightforward solution of the system results in the desired coefficients of the polynomial (2). Figure 1 schematically shows the support configurations we primarily study while the corresponding substrate shapes can be inferred from Table 1 where  $Q_i$  are listed as a function of the supporting lines separation. Note that for  $N = 4$ , the expressions become fairly lengthy, so we only list a particular case when one of the supports coincide with the substrate edge ( $d = W$ ). With the substrate shape  $w(x)$  known, the glass substrate sag  $s$  can be found as

$$s = -\min_{x \in [0, W/2]} w(x) \quad (4).$$



**Figure 1. Schematic of the glass substrate supports (a)  $N = 2$  supports, (b)  $N = 3$  supports, (c)  $N = 4$  supports,  $W$  is the width of the glass substrate.**

To validate this approach, we compared the glass substrate shape due to gravitational sagging calculated from equations (2), (3) and Table 1 with results of FEM calculations (Figure 2). We assume  $E = 7.0967 \times 10^7$  kPa,  $\nu = 0.23$ ,  $\rho = 2.37$  g/cm<sup>3</sup> corresponding to EAGLE2000™\* glass, substrate width  $W = 1850$  mm and thickness  $t = 0.7$  mm. Figure 2 shows very good agreement between numerical and analytical calculations. The clear advantage of the analytical approach is its computational efficiency that becomes especially critical for large-size glass substrates. In addition, sag's dependence on the glass thickness can be straightforwardly obtained from (3) without repetition of the time-consuming FEM calculations.



**Figure 2. Glass substrate shapes  $w(x)$  due to gravitational sagging for  $N=2, 3$ , and  $4$ . For all cases, the distance between the support lines is kept constant and equal 1000 mm. Solid curves: analytical results, dashed curves: FEM calculations assuming  $L = 2200$  mm where  $L$  is the length of the glass substrate along supporting lines ( $y$  axis). Instead of knife-edge supports, the FEM calculation takes 23 discrete supports with equal spacing along the  $y$  axis.**

### 3. Results and Discussion

#### 3.1 Sensitivity to the optimum support position

Obtained analytical results allow for optimization of the position of the supporting lines that minimizes the plate sag [2, 3]. For example, for the case of two supporting lines ( $N = 2$ ), straightforward calculations lead to a cubic equation  $4k_2^3 - 12k_2^2 + 3 = 0$  for the optimum ratio  $k_2 = a_{opt}/W$ ,  $0 < k_2 < 1$ . The only acceptable solution is  $k_2 \approx 0.5537$ , i.e. for

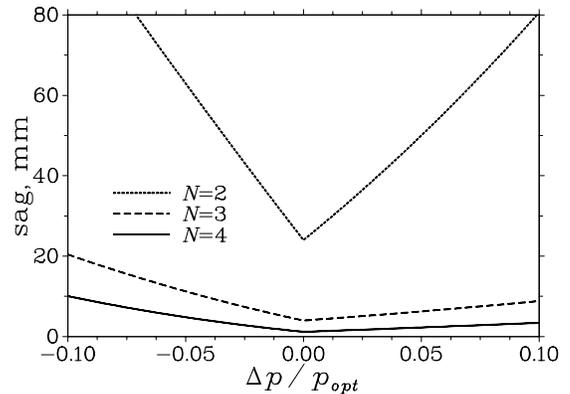
$$a_{opt} = 0.5537 W \quad (5)$$

the glass substrate experiences the minimum sag. Similarly, for  $N = 3$ , the corresponding equation to solve is  $6k_3^4 - 34k_3^3 + 75k_3^2 - 72k_3 + 24 = 0$  ( $k_3 = b_{opt}/W$ ) from where we obtain

$$b_{opt} = 0.7153 W \quad (6).$$

The situation becomes more involved for the case of four supporting lines where two-dimensional optimization needs to be performed. The support positions minimizing the sag are [2]

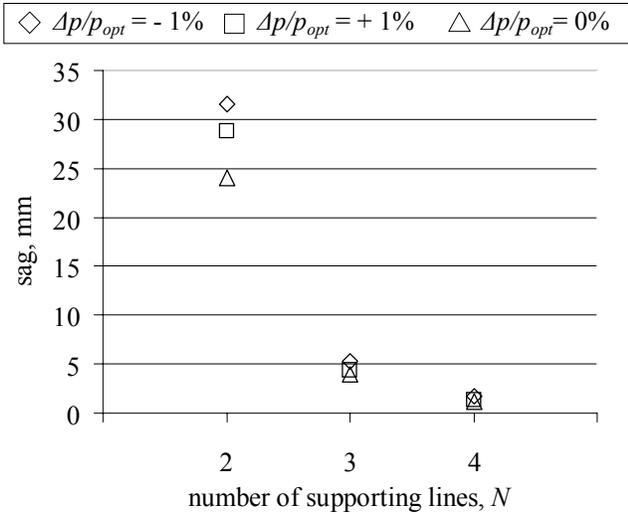
$$c_{opt} = 0.2658 W, \quad d_{opt} = 0.7909 W \quad (7).$$



**Figure 3. Sag variation for the support position deviation from the optimum value  $p = a, b$ , or  $c$  depending on the number of supports and  $\Delta p = p - p_{opt}$  (see Figure 1).**

Figure 3 shows the sag values near the optimum support position. Both the sag magnitude and its sensitivity to support position (slopes of the lines in Figure 3) decrease with increased number of horizontal supports  $N$ .

\*While this paper references a discontinued product, EAGLE2000™, these results apply to any elastic plate that satisfies the requirements of the model.



**Figure 4. Sag variations for the support position deviation by  $\pm 1\%$  from the optimum value of  $p = a, b, \text{ or } c$  and their dependence on number of supporting lines  $N$ .**

$$\Delta p = p - p_{opt}$$

Figure 4 is another illustration of the sag sensitivity to deviation of the support position from the optimum value. For  $N = 2$ , sag variation due to  $\pm 1\%$  support position deviation is more than 5mm. On the contrary, for  $N = 3$  and  $N = 4$ , the same variation of the support position results only in marginal ( $< 1\text{mm}$ ) sag variations. We also note that the sag value itself is drastically decreased in those cases. Figure 4 demonstrates robustness of the glass transportation system with larger number of supporting lines to variation of the positions of supporting lines.

### 3.2 Effect of the initial shape

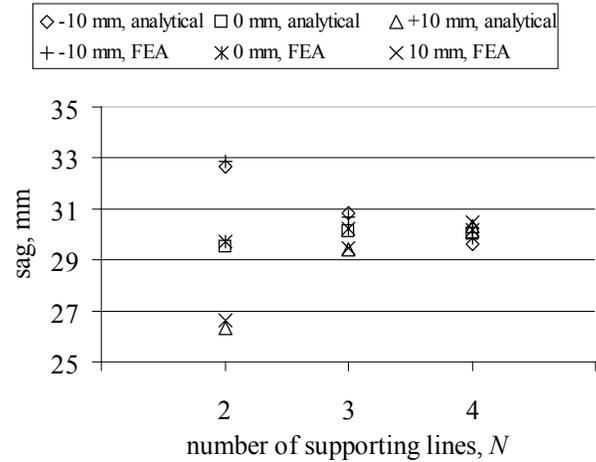
To take the initial shape of the glass substrate into account, we accordingly modify the boundary conditions when calculating coefficients  $Q_i$ . If, for example, for  $N = 3$  the shape deviation at the central support line ( $x=0$ ) is  $w_{initial} = -\delta$ , the modified coefficients  $\tilde{Q}_i$  will be given by

$$\tilde{Q}_i = Q_i + \Delta Q_i \quad (8)$$

where values of  $\Delta Q_i$  are listed in Table 2.

The initial shape is then added to the resulting polynomial given by Table 2. Results of this approach are shown in Figure 5 where we plot the sag value as a function of  $N$ . FEM calculation results are also shown in Figure 5 to verify the accuracy of the analytical formulas. It should be noted that unsupported lengths are selected ( $a = 1036 \text{ mm}$ ,  $b = 1850 \text{ mm}$ , and  $c = 980 \text{ mm}$ ,  $d = W = 1850\text{mm}$ , for  $N=2, 3$ , and  $4$ , respectively) so that the nominal sag for a flat initial shape is approximately 30mm for all cases ( $N = 2, 3$ , and  $4$ ). As can be clearly seen from Figure 5, the increased number of supporting

lines drastically decreases the sag variations even though the nominal sags are similar. The range of sag variations (i.e. sag for  $\delta = -10\text{mm}$  deviation from flat plane minus sag for  $\delta = +10\text{mm}$  deviation from flat plane) decreases by order of magnitude when the number of supporting lines  $N$  changed from 2 to 4.



**Figure 5. Impact of the initial shape of the substrate on the sag variations. Rectangles: flat initial shape, diamonds: convex parabolic initial shape towards the ground, triangles: concave parabolic initial shape towards the ground. EAGLE2000™<sup>1</sup> glass is assumed with the thickness  $t = 0.7 \text{ mm}$  and the width  $W = 1850 \text{ mm}$ ;  $a = 1036 \text{ mm}$ ,  $b = 1850 \text{ mm}$ , and  $c = 980 \text{ mm}$ ,  $d = W$ , for  $N=2, 3$ , and  $4$ , respectively. These parameters correspond to approximately the same sag of about 30 mm for the flat initial shape. The initial glass shape is a symmetric parabola with the maximum variation in the vertical direction of  $\pm 10 \text{ mm}$ .**

### 4. Conclusions

Analytical formulas to calculate sag of the glass substrate are obtained. We introduced a concept of initial shape of glass substrate to simulate sag variations which are caused by various factors. The formulas can deal with three different types of support configurations (number of supporting lines from 2 to 4). It was confirmed that the formulas give accurate results for parabolic initial shapes by comparing with finite element analysis results. Our study revealed that sag sensitivity to the support positions is drastically reduced as number of support lines increases. By introducing initial shape concept, we showed that sag variations caused by various factors are greatly reduced as number of support lines increases. That 90% reduction of sag variations may be possible by increasing number of support lines from 2 to 4 based on the results derived by this study. These results imply that large glass handling can be made more reliable by increasing number of support lines. Glass substrate transportation problems will become more costly as glass substrate generation advances. It was shown that it is

possible to handle large glass substrate reliably by designing good supporting system such as increasing supporting lines. These findings imply that the technological change from solid supports to air-bearing supports may be very advantageous for the next generation of glass substrates.

**References**

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**Table 1 Coefficients of the polynomial (2) for sections I and II as shown in Figure 1**

	$Q_0$	$Q_1$	$Q_2$	$Q_3$
<b>I</b> ( $N=2$ )	$a^2(a^2 - 12aW + 6W^2)/16$	0	$3(2a - W)W/2$	0
<b>II</b> ( $N=2$ )	$a^2(a^2 - 16aW + 6W^2)/16$	$3a^2W/2$	$-3W^2/2$	$2W$
<b>I</b> ( $N=3$ )	0	0	$3(b^2 - 4bW + 2W^2)/8$	$3W - b/4 - 3W^2/(2b)$
<b>II</b> ( $N=3$ )	$-b^2(b^2 - 4bW + 6W^2)/32$	$3b(b^2 - 4bW + 6W^2)/16$	$-3W^2/2$	$2W$
<b>I</b> ( $N=4$ , $d = W$ )	$(c^2/32)(c^3 - c^2W - 9cW^2 + 3W^3)/(2c + W)$	0	$(3/8)(c^3 + c^2W + 3cW^2 - W^3)/(2c + W)$	0
<b>II</b> ( $N=4$ , $d = W$ )	$(-c^2W/32)(c + 3W)(3c^2 - 6cW + W^2)/((c - W)(2c + W))$	$(3c^2/16)(c + W)(c^2 - 5W^2)/((c - W)(2c + W))$	$(3W/8)(-c^3 + 7c^2W + cW^2 + W^3)/((c - W)(2c + W))$	$(c + W)(c^2 - 5W^2)/(4(c - W)(2c + W))$

**Table 2 Perturbations of the coefficients of the polynomial (2)  $\Delta Q_i$  for the case of  $N = 3$  supports**

	$\Delta Q_0$	$\Delta Q_1$	$\Delta Q_2$	$\Delta Q_3$
<b>I</b>	$\delta/R$	0	$-6\delta/(b^2R)$	$4\delta/(b^3R)$
<b>II</b>	$3\delta/(2R)$	$-3\delta/(bR)$	0	0

<sup>1</sup>EAGLE 2000 is an early composition and trademark of Corning Incorporated, Corning, N.Y.

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