Advanced Mounting System for Light Duty Diesel Filter

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ABSTRACT

This paper employs a systematic approach to packaging design and testing of a system and its components in order to determine the long term durability of light duty diesel filters. This effort has utilized a relatively new aluminum titanate filter technology as well as an advanced support mat technology engineered to provide superior holding force at lower temperatures while maintaining its high temperature performance. Together, these two new technologies form a system that addresses the unique operating conditions of diesel engines. Key physical properties of both the filter and the mat are demonstrated through laboratory testing. The system behavior is characterized by various laboratory techniques and validation procedures.

INTRODUCTION

Stringent legislation for diesel exhaust emissions like EURO IV and EPA 2010 is posing significant challenges in terms of higher filtration efficiency, low back pressure, reduced levels of both NOx and particulate matter (PM), and 160,000 km durability for European light duty application [1]. Indeed, improved combustion efficiency via atomized fuel/air mixture (homogeneous charge) and advanced combustion chamber design, along with ultra low sulfur fuel, has helped reduce both gaseous emissions and particulate matter. However, further reductions require the use of an oxidation catalyst (DOC) along with a particulate filter (DPF).

The 160,000 km durability requirement for light duty application requires a robust packaging design, notably for DPF, comprising of innovative materials with proven durability over a wide temperature range. The filter material, for example, must have excellent thermal shock resistance to sustain uncontrolled regeneration events leading to high temperatures. The monolithic, honeycomb aluminum titanate material, with low coefficient of thermal expansion (CTE), high heat capacity, resistance to ash attack, and chemical durability, meets these requirements [2].

Similarly, the support mat wrapped around the filter must be stable over a wide temperature range with excellent cold holding performance and superior erosion resistance at high temperature. This new advanced mat is based in a SiO2-MgO fiber matrix that provides superior holding force at lower operating temperatures. As the system heats-up and thermal gap expansion increases, the small vermiculite content of the product (20.0%) is activated and provides additional holding force at higher temperatures [3]. Ferritic stainless steel alloys with proven performance for both close-coupled and underbody catalytic converters are another durable material for packaging the system. [4-6]

This paper focuses on the properties of advanced filter and mat materials and provides the data for filter assembly simulating real life operation. These data demonstrate that filter durability for light duty applications can be enhanced by proper choice of component materials, characterizing their behavior over the operating temperature range, selecting an optimum gap bulk density, and ensuring that the filter assembly passes accelerated tests designed for long term durability.

PHYSICAL PROPERTIES

FILTER PROPERTIES

Aluminum titanate is a novel ceramic oxide composite with high heat capacity, high melting temperature and a unique microstructure which results in low CTE as well as low elastic modulus (E-mod). These attributes permit monolithic construction of the filter and high regeneration temperatures without compromising the filter thermal shock resistance. [2] The filter has also been shown to be successfully used in vehicle applications. [7]

The desired porosity and pore size distribution for reducing back pressure are achieved by a judicious choice of raw materials and pore formers. Figure 1 is the SEM image of a polished cross-section of the aluminum titanate filter showing well connected
microstructure, which contributes to low back pressure and adequate strength (modulus of rupture, MOR).

Figure 1: SEM Image of Polished Cross-section of Aluminum Titanate Filter Wall Showing Well Connected Microstructure [2]

Table I lists the key physical properties of the aluminum titanate filter.

**Table I: Physical Properties of Aluminum Titanate Filter [2]**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell Density</td>
<td>0.465 cells/mm(^2) (300 cells/in(^2))</td>
</tr>
<tr>
<td>Wall Thickness</td>
<td>0.33 mm (0.013 in)</td>
</tr>
<tr>
<td>Mercury Intrusion Porosity</td>
<td>49-51%</td>
</tr>
<tr>
<td>Mean Pore Size</td>
<td>15-18 (\mu)m</td>
</tr>
<tr>
<td>Bulk Density</td>
<td>0.72 g/cm(^3)</td>
</tr>
<tr>
<td>MOR (axial)</td>
<td>1.47 MPa (213 psi)</td>
</tr>
<tr>
<td>Elastic Modulus (axial)</td>
<td>1.45 GPa (0.21 (\times) 10(^8) psi)</td>
</tr>
<tr>
<td>CTE (25-1000°C) (axial)</td>
<td>9 (\times) 10(^{-7})/(\circ)C</td>
</tr>
<tr>
<td>Minimum Isostatic Design Strength</td>
<td>&gt;2 MPa</td>
</tr>
</tbody>
</table>

While mechanical strength (MOR) is higher for SiC than that of aluminum titanate, an estimate of thermal shock resistance of the filter, defined by thermal shock parameter TSP, is readily obtained by substituting the above properties in Equation 1:

\[
TSP = \frac{MOR/\sqrt{E}}{CTE} = 1127
\]

This value is an order of magnitude higher than that of a SiC filter [2]. Alternatively, since thermal stresses during regeneration are proportional to the product of CTE and E-mod, the low CTE and E-mod of the aluminum titanate filter help minimize these stresses, thereby ensuring good thermal durability.

The well connected pore structure of the aluminum titanate filter leads to low pressure drop during filtration as demonstrated in Figure 2. A comparison of clean and soot loaded pressure drop of aluminum titanate and SiC filters demonstrates similar pressure drop in the clean state and better performance of the aluminum titanate filter in the soot loaded case. [7]

Figure 2: Pressure Drop Curves for an Aluminum Titanate Filter vs. SiC Filter [7]

The chemical durability of an aluminum titanate filter was measured by exposing it to i) engine ash (containing Fe, P, Ca, Zn, etc.), ii) iron and iron oxide (debris in the exhaust system), iii) high temperature oxidizing and reducing conditions and iv) acidic solutions resulting from the reaction of SO\(_2\) and H\(_2\)O in the presence of catalyst. While the results of these tests are described elsewhere [2], it suffices to conclude that the aluminum titanate filter passed all of the durability tests in most severe environment with no evidence of chemical reaction or decomposition.
SUPPORT MAT PROPERTIES

The advanced support mat described in this study is a third generation system that incorporates several performance improvements such as excellent cold holding performance, thermal stability and erosion resistance. The combination of performance features described above allows this mat to be installed without the need of additional heat-treatment typically associated with traditional intumescent mats or the use of end seal metallic rings for erosion protection [8].

The ceramic fiber matrix used in this product also complies with both European Directive 97/69/EC and German legislation that establishes guidelines for the use of true green products when it comes to work safety and vehicle component recycling [9-11].

Support Mat Aging

In service, support mats will age due to thermal and mechanical influences. The main thermal factor in diesel emission control devices is the overall low operating temperature associated with high g-loads. The mechanical factors include relaxation due to fiber rearrangement and may also include fiber breakage. The main cause of mechanical action on the mounting mat is gap expansion and contraction driven by the thermal expansion properties of substrate and shell.

When a converter is at ambient temperature, the gap between the shell and substrate is under stable conditions. Once the system is exposed to heat, the shell begins to expand more rapidly than the substrate resulting in “gap expansion” and creating a larger gap. If improperly designed, a system will lose mat pressure and erosion will compromise substrate durability. The support mat system must be able to absorb the increased gap and provide sufficient pressure to maintain system integrity.

Several different aging tests can be performed to measure support mat performance. The "1000 Cycle Test" [12] is a standard aging test that has been successfully used over the last decade as a robust test for support mat accelerated aging. The test is designed to simulate mechanical cycling associated with expansion and contraction of the converter shell. The expansion and contraction of the shell are indicative of conditions likely to be experienced during the life of a converter as a result of thermal cycling in day-to-day use. Figure 3 exemplifies the testing apparatus used to measure aged mat pressure.

![Figure 3: Low Temperature Aged Mat Pressure](image)

Cold Holding Performance

Traditional expanding mats rely on vermiculite thermal expansion to generate the holding force required by the system. Therefore, a traditional expanding mat is not likely to perform properly in DPF applications without the aid of heat-treat and metallic seals as the typical inlet gas temperature is below the minimum required for vermiculite expansion [13-15].

This new advanced support mat relies on a spring-like fiber matrix that generates the required holding force while the system is operating at low temperatures. In that sense, the advanced support mat will present a behavior that is similar to fiber-only support mats. Figure 4 presents the aged mat pressure for different support mat types at 350°C. Notice that no previous heat-treatment has been made to any mat in this study.

![Figure 4: Low Temperature Aged Mat Pressure](chart)
Hot Holding Performance & Thermal Stability

Diesel particulate filters must go through a regeneration process in order to eliminate the soot accumulated during regular engine operation. At this point, a large amount of heat will be generated and the system’s temperature will rise. Since the aluminum titanate filter has a very low thermal expansion coefficient, the relative gap expansion of the metal shell will be significant. In this case, the support mat must be capable of absorbing the gap expansion while maintaining sufficient holding force.

Traditional expanding mats will present good performance under this condition since there is enough temperature to activate the vermiculite particles. Fiber-only mats still present good performance but performance is directly affected by gap expansion.

The advanced support mat used in this study presents a more stable performance over different temperature ranges. When the system operates at higher temperatures, the low vermiculite content present in the product will compensate the effects of gap expansion. Figure 5 presents the aged mat pressure for different mats at 900°C.

![Figure 5: High Temperature Aged Mat Pressure](image)

**Erosion Resistance**

Support mat erosion is another failure mechanism that may occur during the life of the emission control device. Erosion is caused by the inability of a design to maintain targeted gap bulk density within the proper design range resulting in low support mat pressure. If a design is unable to manage the gap bulk density either due to shell deformation or if the temperature of the converter causes shell expansion to be too great for the mat to absorb, erosion will result.

In such cases, support mats are susceptible to two main erosion mechanisms. The first one is the product ability to withstand direct gas impingement on the exposed edge. The second one is the resistance of the fiber matrix against indirect drag forces caused by vacuum zones formed inside or close to the inlet cone region.

There are several different methods used to protect the support mat against erosion. The most common techniques include the use of end seal rings or edge treatment of the support mat. These mechanisms are likely to add extra cost to the system and reduce productivity.

Fiber-only mats present excellent erosion resistance and are considered the benchmark in terms of performance. It is possible, however, to measure the relative volume loss of different support mats installed at the same gap.

Erosion resistance can be quantified by using a test apparatus designed to simulate a four cylinder engine running at 6000 rpm [16]. Samples from different mat types are then mounted in adjustable fixtures as shown in Figure 6.

![Figure 6: Erosion Test Apparatus](image)

The test fixtures are then closed to a specified GBD and aged twice to 600°C. The samples are subsequently cooled to room temperature. The edges of the mat, while under compression in the fixture, are subjected to the pulsing room temperature air-stream for 50 minutes. Mat volume loss caused by the air-stream attack is then calculated and provides a relative measure of mat durability.

Figure 7 describes the erosion resistance of different support mats over a typical GBD window.
The advanced support mat has an erosion resistance that is far superior to that of traditional expanding mats. Due to the high fiber content, its erosion resistance is very similar to a fiber-only product provided it is installed and maintained at its nominal GBD range.

**LABORATORY TEST DATA**

**Defining Minimum Application Requirements**

Design of a packaging system requires analysis to determine the minimum required shear strength the package needs to withstand for a specified axial acceleration (assuming amplitude ratio is negligible) and exhaust gas pressure drop forces across the filter. A durable packaged system has a shear strength that is greater than the sum of the forces acting in the axial direction (typically acceleration and exhaust gas pressure drop) divided by the interface area between support mat and filter, as shown below:

\[
\tau_u = \frac{\Delta P \cdot A_{CS} + m \cdot a}{A_m}
\]  

(2)

Where:

- \(\Delta P\) [kPa] Pressure drop across the filter
- \(A_{CS}\) [m²] Cross sectional area of the filter
- \(m\) [kg] Filter mass
- \(a\) [m/s²] Axial acceleration
- \(A_m\) [m²] Support mat area

Appendix I presents the necessary equations for the minimum required shear strength calculation.

Based on the parameters of this study, the required system shear strength can be determined for the aluminum titanate filter. Generalized design numbers will be used so that confidentiality of design information is preserved.

**Table II: Example Application Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Aluminum Titanate DPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta P)</td>
<td>60 kPa</td>
</tr>
<tr>
<td>(A_{CS}) (\text{cm}^2)</td>
<td>170.2 cm²</td>
</tr>
<tr>
<td>(m) (\text{kg})</td>
<td>2.1 kg</td>
</tr>
<tr>
<td>(a) (\text{m/s}^2)</td>
<td>490 m/s²</td>
</tr>
<tr>
<td>(A_m) (\text{cm}^2)</td>
<td>870.8 cm²</td>
</tr>
<tr>
<td>Required Shear Strength</td>
<td>23.5 kPa</td>
</tr>
</tbody>
</table>

**Gap bulk density - Canning Design Window**

Support mats behave as visco-elastic systems and can be schematically represented by a spring and a damper [17]. Upon compressing the material, a large compressive resistance (peak pressure) is produced. As the damper relaxes over time, a lower compressive resistance (residual pressure) is produced according to the spring constant of the system. Figure 8 shows the classic representation of a visco-elastic system.

![Figure 8: Typical Visco-Elastic System](image)

During canning the entire system is subjected to the peak pressure corresponding to the installed GBD. The peak pressure must be kept below the strength of the filter in order to prevent filter breakage. Therefore, GBD is a key process control variable.

After the canning process is completed, the mat pressure gradually decays until it reaches a constant level. In the visco-elastic material model, this is analogous to the damper relaxing. Once the relaxation is complete, the residual pressure is reached.

In service, the pressure decays further due to thermal and mechanical influences. The main thermal factor is viscous creep of structural fibers in the mat causing the structure to relax. The mechanical factors include relaxation due to fiber rearrangement and may also include fiber breakage. The main cause of mechanical action on the mounting mat is gap expansion and contraction driven by thermal expansion properties of filter and shell. A robust canning process must assure that residual pressure in the system will be high enough to withstand all aging factors described above.

**Canning procedure**

Aluminum titanate oval filters were canned via a standard tourniquet method [18] using a 2700 g/m² basis weight mat and a 16 gauge, 409 stainless steel can with a stamped recess for the lap joint. The filter-mat-can assembly was placed in a tourniquet type strap which...
was securely fastened in a 20,000 lbf load frame. Since the filters are oval in shape, a “plugged” strap was used [18] to help maintain constant compression around the azimuth direction of the filter face.

During the strap tightening process, the force exerted by the load frame was incrementally increased as gap measurements were taken at eight points around the perimeter of each face of each filter until the target gap was reached, at which time the load frame force was recorded (Figure 9). Subsequent runs closed the assembly to the same tourniquet force as this initial calibration run with periodic gap and GBD checks to confirm consistency.

![Figure 9: Numbered End Face of Canned Filter](image)

### Canning Results

All aluminum titanate filters were canned to a slightly lower than nominal target gap bulk density of 0.61 g/cm³ (4.4 mm target gap.) Gap measurements were taken after the can was welded shut. Table-III shows a summary of the average gap range (part to part comparison).

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Target:</td>
<td>4.4 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average:</td>
<td>4.10 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min:</td>
<td>3.90 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max:</td>
<td>4.51 mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Intrapart gap variability measurements showed relatively tight gaps near the overlap joint of the can, which is a common occurrence in tourniquet canning of oval shaped filters.

Assuming the nominal basis weight of 2700 g/m², the average installed GBD was close to 0.66 g/cm³.

### Measuring Peak and Residual Pressure

Tekscan is a commercially available thin film pressure sensor which allows the user to monitor pressure fields in real time. During the canning process, a sensor is wrapped around the filter, which in turn is covered by the mat material and put into the canning process. The pressure distribution during canning is recorded in real time, allowing the measurement of both peak and residual pressure [19].

Figure 10 presents the canning pressure profile for aluminum titanate filters wrapped with the advanced support mat system.

![Figure 10: Tekscan Results (4.4 mm target gap)](image)

As a key to the reader, the Tekscan plots may be interpreted as a standard contour plot where dark color indicates a high pressure spot and light color indicates lower pressure regions. For each test the y-axis represents the filters axial length and the x-axis is wrapped around the perimeter of the filter and represents an unwrapped version of the filter skin pressure.

All three tests showed a very similar peak pressure pattern and distribution. The results can be summarized by:

- Peak pressure range: 1,862 – 2,068 kPa.
- Average pressure range: 441 – 462 kPa.

As seen in most of the oval tourniquet processes, the pressures at the major axis opposite to the lap joint showed the lowest pressure values, 138 – 483 kPa, since this region naturally presents higher gaps resulting in less mat compression.
When correlating these Tekscan average pressures to pressures from the GBD compression curve it is important to note that the Tekscan samples were not tack welded and taken out of the tourniquet strap. Consequently one would expect the pressures to further decrease due to elastic “spring back” of the can post removal from the tourniquet strap.

If we use the Tekscan average pressures to find the average GBD, we find that 448 kPa is equivalent to a GBD of 0.63 g/cm³.

Similar tourniquet canning was performed on round, 4.16”Ø x 3.00”L cordierite substrates (350cpsi/5.5 mil wall) for Resistive Thermal Exposure (RTE) testing (described in a later section of this report). A gap bulk density of 0.61 g/cm³ was also targeted for these products.

MOUNT DURABILITY

RTE TESTING

Resistive thermal exposure (RTE) testing is an accelerated thermal aging test of the complete packaging system (aging is carried out on a canned assembly in which a temperature gradient across the mat material is induced and cyclically aged; the filter matrix material is not aged in this type of test.) The system aging is then followed by a hot axial push out test which allows for calculation of shear strength of the packaged system. Numerous references are available for a more detailed description [20].

Following canning, the parts were prepared for RTE testing. This requires kanthal wires to be inserted through the cells of the filters (around the perimeter), and subsequent spot welding to form a continuous electrical circuit (Figure 11).

Each filter had to be wired in two independent circuits (Figure 11) which were then connected in parallel to reduce the resistance of the total heating element due to temperature controller limitations. The parallel wiring scheme allowed the test piece to be brought up to test temperature at an acceptable rate typical of RTE cycling.

A voltage/current was applied to the wires which in turn resistively heated the filter skin. A thermocouple was inserted into the cell closest to the filter skin. A programmable controller was used to manage this thermocouple temperature to a prescribed thermal profile. A picture of the final test set up is shown in Figure 12.

Once these parts were thermally aged (cycled 400 times), the final step was to apply a load to the filter in the axial direction and measure residual shear strength of the packaged system (Figure 13).
The filter was placed in a load frame and the RTE wires were connected to a temperature controller. The filter skin temperature was heated to its aging temperature and held for several minutes until the system reached a steady state condition at which time the load was applied. The displacement controlled load was applied to the filter totaling 3.0 mm of filter travel. Force was monitored during the push out test. Shear strength was then calculated by dividing the maximum force obtained during the push out test by the mat-filter contact area.

**RTE RESULTS**

Results from the tourniquet canned (16ga, 409 stainless steel), 4.16”D x 3.0”L, round cordierite substrates are shown in Figure 14. A strength increase was observed after aging at substrate skin temperatures of 450°C and less. At temperatures greater than 450°C, an initial peak strength relative to vermiculite expansion will occur, but it levels off gradually as aging occurs.

**RTE Test Results**

Advanced Support Mat @ 2700 g/m² Basis Weight and 0.61 g/cm³ GBD

The unique advantage of this advanced support mat can be illustrated when the RTE results in Figure 15 are compared with those of a standard intumescent mat. Figure 15 shows the significantly enhanced shear strength at low temperatures (4.7X) compared to that of the standard intumescent mat after 400 thermal cycles. This is the key to enhanced performance in low temperature applications such as diesel exhaust systems while the low content of vermiculite helps increase performance at higher temperatures.

**Advanced Support Mat vs. Standard Intumescent**

RTE Test Results - Ø4.16” x 3”, 350/5.5 cordierite substrate, 16ga 409 Stainless Steel, Tourniquet canned

From the results given in Figure 14 and Figure 15, the 280°C filter skin temperature provided the lowest system strength. Six aluminum titanate oval filters were tested at this skin temperature (3 with 0 thermal cycles and 3 with 400 thermal cycles). Figure 16 shows the consistent trend of increasing strength with increasing cycles at low temperatures. The lowest strength for this design was measured to be approximately 38 kPa (5.5 psi).
The RTE results of the aluminum titanate oval yielded strengths which exceeded the calculated required strength by a factor of 1.6X - 2.2X (assuming 490 m/s² acceleration, 60 kPa peak pressure drop & 2.1kg weight – see appendix) thereby suggesting the design is durable at the specified conditions.

HOT VIBRATION STUDY

A hot vibration test was also conducted at the nominal GBD to evaluate system durability. The hot-shake test was performed with an aluminum titanate filter (oval); the following protocol was used:

- Sine Wave: 190 Hz
- Axial acceleration: 735 m/s² (75g)
- Inlet gas temperature: 250°C
- Number of cycles: 450
- Type of cycle: 15 minutes of heat + 20 seconds of water quench

The system was checked at periodic intervals during and at the conclusion of the test - at which time the filters location was measured to check for relative filter-mat-can movement. Results indicated no relative change in position of the filter at the beginning and end of the test. The mat was also visually inspected for any signs of erosion and/or other obvious damage – none was seen. In all cases the filters were intact and the system maintained its integrity.

CONCLUSION

The design of diesel emission control devices presents a challenge to current mounting systems due to its unique operating conditions and high durability requirements. A systematic approach to system design and testing is a key element for robust performance and was successfully completed for the aluminum titanate filter – advanced support mat system.

The robustness of the aluminum titanate filter, advanced support mat and ferritic stainless steel can mounting system was established for this specific light duty diesel application. This was accomplished through mat coupon testing, RTE results, and hot vibration verification coupled with modeling results.

RTE results on round, cordierite substrates demonstrated that the advanced support mat maintained a 4.7X strength advantage over a standard intumescent mat after 400 thermal cycles of system aging at 280°C substrate skin temperature; all while continuing to increase strength at skin temperatures in excess of 450°C. This also confirms the mat coupon testing general trends.

The RTE testing performed on the aluminum titanate oval DPF with an advanced support mat mount system showed a 38 - 51 kPa shear strength from 0-400 cycles and 280°C filter skin temperature. Comparing this measured strength to the example calculated minimum required shear strength of 23.5 kPa indicates a durable system for the stated conditions.

Further, the system was validated on a hot vibration test at more than 735 m/s² acceleration and at low temperatures for many cycles providing a further validation of system and component durability.

The presented mounting system is in compliance with the European Directive 97/69/EC, German legislation and also with the North American legislation [9-11].

ACKNOWLEDGMENTS (CORNING & UNIFRAX)

The authors would like to thank Mr. John Ten‘Eyck and Mr. Lars Findler at the Unifrax Corp for their commitment to this development and for the precious review work provided to this team. We would also like to thank Dennis Edwards at Corning Inc. for performing the RTE testing reported in this paper.

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APPENDIX – REQUIRED SHEAR STRENGTH CALCULATION

In mathematical terms:

\[ \tau_u = \frac{1}{A_m} \sum_{i=1}^{n} F_i \]  

(1)

where \( F_i \) represents each component of axial force acting on the substrate/filter (\( i \) is a free variable), “\( A_m \)” is the area which the mat covers the filter and \( \tau_u \) is shear strength required to resist the applied forces.

The individual force components (\( F_i \)) can be calculated via the following:

\[ F_p = \Delta P \cdot A_{CS} \]  

(2)

\( F_p \sim \) Axial force due to exhaust pressure drop
\( \Delta P \sim \) Exhaust pressure drop across the filter/substrate
\( A_{CS} \sim \) Cross sectional area of the filter/substrate

and,

\[ F_a = m \cdot a \]  

(3)

\( F_a \sim \) Axial force due to mechanical acceleration
\( m \sim \) filter/substrate mass
\( a \sim \) axial acceleration of filter/substrate

Assuming these are the only two axial forces on the system, equations (2) and (3) can be substituted in equation (1) yielding,

\[ \tau_u = \frac{\Delta P \cdot A_{CS} + m \cdot a}{A_m} \]  

(4)

Given nominal geometrical design values of the filter cross sectional area, mat coverage length and mass, the minimum required shear strength can be determined as a function of acceleration and pressure drop.

Assuming the following parameters, a required shear strength can be found.

Table IV: Example Application Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P )</td>
<td>60 kPa</td>
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<tr>
<td>( A_{CS} )</td>
<td>170.2 cm(^2)</td>
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<tr>
<td>( a )</td>
<td>490 m/s(^2)</td>
</tr>
<tr>
<td>( A_m )</td>
<td>870.8 cm(^2)</td>
</tr>
</tbody>
</table>

\[ \tau_u = \frac{(60000Pa) \cdot (0.01702m^2) + (2.1kg) \cdot (50g) \cdot \left( \frac{9.8 \frac{m}{s^2}}{1g} \right)}{0.08708m^2} \cdot \frac{1kPa}{1000Pa} \]  

(5)

As shown, the RTE results show strengths which exceed the calculated required strength to survive the example application loads and thus suggest the design is durable at the specified conditions.