Statistical PMD Specification – Evolution, Utilization and Control

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Abstract: Polarization mode dispersion is a primary consideration in today's telecommunications networks as it can be an important limiter of high data rate and long reach systems. The effective polarization mode dispersion attributed to cabled optical fiber in such systems is dependent *not* solely upon the maximum individual PMD value, but instead upon the link PMD which is dictated by the PMD distribution of all cabled fibers comprising the link. Taking advantage of the statistical specification of polarization mode dispersion allows one to maximize system efficiency and eliminate unnecessary conservatism. Presented are methods utilized to specify and control PMD which can have a beneficial effect on the cabling process as well as on system design and performance.

OCIS codes: (060.2400) Fiber properties; (060.2280) Fiber design and fabrication; (060.2360) Fiber optics links and subsystems

1. Introduction

The fundamental mode of single-mode fiber operation consists of two orthogonal polarization states which experience a relative offset as they propagate. The magnitude of this offset in the time-domain is termed the differential group delay (DGD), which varies randomly as a function of time and wavelength. This statistical variation of DGD is described by a Maxwell probability density function, with the mean of the Maxwell distribution termed the polarization mode dispersion (PMD) value. For fibers which experience random coupling of the orthogonal polarization states this value can be further normalized by the square root of the fiber length, resulting in the PMD coefficient. Similarly to chromatic dispersion, the implication of PMD is pulse-broadening, which can result in bit errors at the receiver. PMD is a potential limiting factor of high data rate and long length systems, making it a necessary candidate for proper specification and control.

In practice, the vast majority of applications which are potentially limited by PMD require the concatenation, or joining, of a number of cabled fibers to produce a link. Historically, the most basic approach to system design has been to assume that each cabled fiber comprising the link has a PMD coefficient equal to the maximum specified value. This "worst case" assumption does not account for the true distribution of PMD coefficients which comprise the link, and has been shown to produce excessively conservative system limitations[1]. An alternate and now widely accepted approach takes advantage of the entire distribution of cabled fibers which may be randomly joined to form a link, providing a more realistic view of actual performance. This approach has been supported by many groups in the optical communications industry and has been incorporated into international standards, including those of the International Electrotechnical Commission (IEC) and the International Telecommunications Union – Telecommunication Standardization Sector (ITU-T)[2-4].

While the statistical specification approach provides a more realistic means of assessing performance, it is subject to impact from both the general form of the distribution of cabled fiber PMD coefficients as well as any outliers of that distribution. These factors can have a substantial impact on the statistical specification results and therefore on system design limitations. Control of outliers and the overall distribution of PMD coefficients can be achieved through reducing asymmetries in the fiber refractive index and stress profiles and through introducing controlled polarization mode coupling via fiber spinning. This paper presents arguments and examples in support of the statistical specification of PMD and discusses its evolution, utilization and control.

2. Specification Evolution

As mentioned previously, the historical approach to system design often involved the assumption that each cabled optical fiber comprising a concatenated link had a PMD coefficient equal to the maximum specified value. By examining the formula:

$$PMD_{link} = \left[\frac{1}{L_{link}} \sum_{i=1}^{N} L_i x_i^2 \right]^{1/2} \qquad (1)$$

where PMD_{link} is the PMD coefficient of the link, L_{link} is the length of the link, N is the number of cabled fibers comprising the link, L_i is the length of the i^{th} cabled fiber and x_i is the PMD coefficient of the i^{th} cabled fiber, one can see that this assumption simply results in the PMD coefficient of the link being equal to that of the cabled fiber PMD coefficient upper specification limit (regardless of the number or lengths of cabled fibers utilized in the link). One could then calculate the total PMD of the concatenated link by multiplying the link PMD coefficient by the square root of the link length. To account for the random variation in DGD as a function of time one would further need to multiply the total PMD by a Maxwell adjustment factor, the value of which is dependent upon the maximum probability at which the DGD budget may be exceeded.

As an example, consider a system with the characteristics summarized in Table 1.

Table 1. Characteristics of a hypothetical optical communications system.

Transmission Rate	40 Gb/s
Modulation Format	RZ, 33% duty ratio
Link Length	1280 km
Individual Cable Length	5 km
Number of Individual Cable Sections Comprising Link	256
Total Maximum System DGD	11.5 ps
Allowable Probability of Exceeding Total Maximum System DGD	1.3×10^{-7}
Maximum DGD due to Optical Fiber Cable	7.5 ps
Upper Specification Limit for PMD Coefficient of Cabled Fiber	0.2 ps/√km

Considering only the contribution due to cabled optical fiber, the conventional "worst case" approach would tell us that the PMD coefficient of the link is 0.2 ps/√km and the total link PMD value (0.2 ps/√km) × (√1280 km) or 7.2 ps. Assuming that the maximum probability of exceeding the total maximum system DGD is shared equally between fiber and components (6.5×10^{-8} each) and thus using a Maxwell adjustment factor of 3.775 to account for the random variation of DGD, the maximum instantaneous DGD is calculated at over 27 ps (again, due only to cabled optical fiber). This approach clearly indicates that the link does not meet the target objective, and that it could only do so if the maximum allowable DGD of the system was increased substantially. In other words, the contribution to total system DGD from cabled fiber alone makes the system inoperable at the desired availability level and allows no room for other system components such as optical amplifiers and dispersion compensation modules.

Standards organizations and participants in the telecommunications industry have acknowledged that the above method does not accurately reflect the actual PMD in a system link, and that it imposes overly conservative system design limitations. As a solution, it was proposed that a statistical approach to PMD specifications be utilized, whereby numerous possible combinations of cabled fiber concatenations are taken into account.

One available method introduced by the IEC and ITU-T, commonly referred to as "Method 1," establishes an upper bound for the PMD coefficient of the concatenated link known as PMD_Q or the link design value. This value represents some cumulative percentile (typically 99.99%) of the link PMD coefficient probability density function. The method can be executed utilizing any of several calculation techniques, including both numerical and analytical means. The parameters of the PMD_Q calculation include M, which represents the number of concatenated cabled fiber sections, and Q, which is equal to 1 minus the cumulative percentile at which the upper bound is located (i.e., 0.01% for the 99.99th percentile). ITU-T G.652, which provides recommendations for the attributes of non-dispersion-shifted single-mode fiber, requires a maximum PMD_Q value of either 0.5 or 0.20 ps/√km using M=20 and Q=0.01% depending upon which of the four product types is considered (i.e., G.652.A, G.652.B, G.652.C or G.652.D). Recommendations for the maximum PMD_Q of specific cutoff-shifted or dispersion-shifted single-mode optical fiber products are also given in the appropriate ITU-T standards.

The IEC and ITU-T introduced "Method 2" as a complimentary approach which lends itself more to system design than specification compliance. This method similarly utilizes the averaging which occurs during the concatenation process to generate a large distribution of possible link PMD coefficients. To reflect the random variability in DGD over time, each link PMD coefficient comprising the distribution is then convolved with a Maxwell distribution to determine the probability of that specific link exceeding a maximum DGD value. This is done for each link PMD coefficient, and the total probability of exceeding the maximum DGD value is normalized by the quantity of link PMD coefficients. This convolution process takes advantage of the fact that the Maxwell distribution is defined by a single parameter, and therefore by knowing the value of that parameter (that is, the PMD coefficient) one can calculate the entire distribution of possible DGD values.

3. Specification Utilization

Consider the same 40 Gb/s, 1280 km system defined by Table I which requires that the end-to-end DGD contribution from optical fiber cable not exceed 7.5 ps at a probability greater than 6.5×10^{-8}. If one assumes a measured distribution of individual cable PMD coefficients as shown in Figure 1, then how does the statistical approach compare to the "worst case" scenario described above?

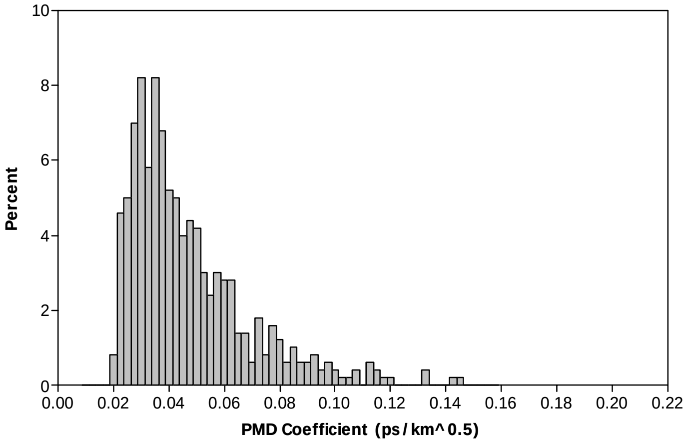

Fig. 1. A simulated distribution of measured cabled fiber PMD coefficients. This distribution was simulated by randomly generating 500 data points from a Gamma distribution with a shape parameter of 1.5, a scale parameter of 0.0175 ps/√km and a threshold parameter of 0.02 ps/√km.

As shown in Figure 2, the statistical upper bound (PMD_Q) of the concatenation of 256 five kilometer cable sections resides at a PMD coefficient of 0.058 ps/√km, far from the 0.2 ps/√km value assumed in the historical approach. This clearly illustrates the beneficial averaging which occurs when adding a distribution of cabled fiber PMD coefficients in quadrature as occurs in an actual system link.

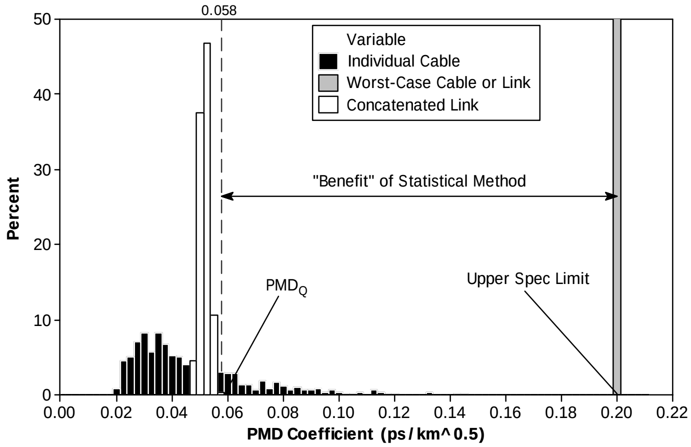

Fig. 2. A comparison of the PMD coefficient distributions arising from the historical "worst-case" assumption and from the statistical concatenation of individual cabled fiber sections. Clearly the upper bound (99.99th percentile), or PMD$_Q$, of the concatenated link distribution (represented by the dashed vertical line) is far below the value obtained using the worst-case assumption. Note that the worst-case distribution continues to a value of 100 percent on the y-axis scale.

Assuming that the system contained random as well as embedded deterministic components, one could utilize the following equation to calculate the total maximum PMD value of the system[3]:

$$PMD_{tot} = \left[L_{link} PMD_Q^2 + \sum_i PMD_{R_i}^2 + \sum_j PMD_{D_j}^2 \right]^{1/2} \qquad (2)$$

where PMD_{tot} is the total system PMD value, L_{link} is the length of the link, PMD_Q is the statistical upper bound on the link PMD coefficient distribution, PMD_{R_i} is the PMD of the i^{th} random optical component and PMD_{D_j} is the PMD of the j^{th} embedded deterministic optical component.

Further illustrating the power of analyzing PMD performance in a statistical manner, utilizing the Method 2 approach and convolving each link PMD coefficient with a Maxwell probability density function results in a probability of exceeding the 7.5 ps DGD target of 8.8×10^{-9}, which is favorable with respect to the 6.5×10^{-8} target. To extract the most from the Method 2 system design approach, one could further utilize the following equation for several purposes. Note that this equation also assumes that all deterministic components are embedded, such that the DGD at a given wavelength is time-randomized by downstream fiber[3]:

$$DGD\max_{tot} = \left[DGD\max_F^2 + S^2 \left(\sum_i PMD_{R_i}^2 + \sum_j PMD_{D_j}^2 \right) \right]^{1/2} \qquad (3)$$

where $DGD\max_{tot}$ is the total maximum DGD of the system, $DGD\max_F$ is the DGD due to concatenated cabled fiber, S is the component Maxwell adjustment factor (e.g. 3.775) which is related to the allowable probability of exceeding the maximum system DGD due to component contributions, and PMD_{R_i} and PMD_{D_j} are the same as defined in Equation (2).

For example, if one found that the above cabled fiber PMD coefficient distribution (shown in Fig. 1) could meet the 6.5×10^{-8} requirement at a DGD target of 6.5 ps (as opposed to the original 7.5 ps target), then the surplus DGD could be utilized for the component budget. Assuming that 12 random and 24 embedded deterministic components are present in the system, Equation (3) would indicate the ability to increase the allowable maximum PMD of each component from ~0.38 to ~0.42 ps.

While the PMD_Q value resulting from Method 1 is typically used for specification compliance purposes and the probability of exceeding a maximum system DGD value arising from Method 2 is typically used for system design purposes, it should be noted that the standards do provide a method whereby near equivalence of the methods can be established[3]. This allows one to ensure that a reference system can meet Method 2 requirements by ensuring compliance with Method 1 and examining the mathematical average of the numerous possible link PMD coefficients. Establishing a region of near equivalence also facilitates the proper use of Equations (2) and (3).

4. A Case Study

The following case further illustrates the application of the statistical approach to network system design as it pertains to PMD.

In 2002, a Chinese cabler manufactured loose tube cables using Corning® SMF-28e® and LEAF® optical fibers. For a 40 Gb/s NRZ network system, the ITU-T proposes a maximum end-to-end system DGD of 7.5 ps for the system assuming a 1 dB cable PMD penalty [5]. If the contribution to DGDmax from cabled optical fiber is restricted to 5.0 ps, then 1.5 ps is available for components in the system assuming that the allowable system failure probability is shared equally between cabled fiber and components (6.5×10^{-8} each).

Utilizing Method 1, the PMD_Q of the cabled fiber was calculated to be 0.079 and 0.045 ps/√km for SMF-28e and LEAF, respectively. Utilizing Method 2 for the 400 km ITU-T reference system comprised of 10 km cables, the system outage probability was calculated to be 7.4×10^{-11} and $< 10^{-20}$ for SMF-28e and LEAF fiber cables, respectively. These results indicated that both cable types would be very reliable for the network system as compared to the maximum allowable outage probability of 6.5×10^{-8} due to cabled optical fiber.

While the above calculation is valid for a 400 km reference system, a more typical non-regenerated link length for national and provincial long haul trunk line networks in China is 800 km, with an amplifier spacing of 100 km and cable lengths of 2 km. When the Method 2 calculation was tailored to the specifics of this system design, the probability of exceeding the maximum allowable system DGD due to cabled optical fiber was determined to be $2.5\text{x}10^{-8}$ and $8.3\text{x}10^{-17}$ for SMF-28e and LEAF fiber cables, respectively. This demonstrated that both cables would again achieve the desired system availability target, with LEAF fiber cable outperforming SMF-28e in terms of a lower probability of outage.

Table 2 presents the above scenarios as Case 1 and Case 2, and also demonstrates another useful application of Method 2 via Case 3. In this scenario, if the system designer wishes to tighten the DGD contribution of cabled fiber from 5.0 to 3.5 ps in order to save more DGD margin for components or more design flexibility, then he could determine that the LEAF® fiber cable can still perform reliably in the system with a low outage probability of $1.8\text{x}10^{-8}$. Additionally, the component maximum PMD could be increased from 1.5 to 1.8 ps. This provides a 20% greater DGD budget for components, which could enable component and system designers to realize significant advantages in design flexibility and cost savings.

The PMD statistical approach also provides important processing information to cabler. In this case, the cabler would observe a difference between cable PMD_Q and uncabled fiber PMD_Q, i.e. cable PMD_Q of 0.079 ps/√km vs. fiber PMD_Q specification of 0.06 ps/√km for the SMF-28e® case, and cable PMD_Q of 0.049 ps/√km vs. fiber PMD_Q specification of 0.04 ps/√km for the LEAF® case. This difference may indicate that there is room for the cabler to improve their cabling processes to achieve lower cabled fiber PMD_Q and to potentially be in a position to offer more competitive specification values.

Table 2. System characteristics and calculated outage probabilities

	Case 1	Case 2	Case 3
Reference Link Length	400 km	800 km	800 km
Reference Cable Length	10 km	2 km	2 km
Transmission Rate	40 Gb/s	40 Gb/s	40 Gb/s
Total DGDmax, 1dB Penalty, NRZ	7.5 ps	7.5 ps	7.5 ps
total cable DGDmax	5.0 ps	5.0 ps	3.5 ps
total component DGDmax	1.5 ps	1.5 ps	1.8 ps
each component unit DGDmax	0.5 ps	0.5 ps	0.6 ps
System Outage Probability			
SMF-28e Cable PMD_Q, 0.079 ps/√ km	7.4×10^{-11}	2.5×10^{-8}	2.7×10^{-4}
LEAF Cable PMD_Q, 0.045 ps/√ km	$<10^{-20}$	8.3×10^{-17}	1.8×10^{-8}

5. Specification Control

To lower the PMD_Q value, one needs to improve the PMD distribution of manufactured fibers. This requires reducing the individual fiber PMD to produce a tighter distribution with a lower average PMD value. Two methods that are commonly used to reduce fiber PMD are (1) reducing fiber intrinsic birefringence and (2) fiber spinning. The first method involves fiber manufacturing process improvements aimed at minimizing asymmetries in the refractive index and stress profiles [6-9]. The second method involves intentional rotation of the fiber birefringence axis during the fiber draw prcess to introduce controlled polarization mode coupling [10-25]. Fiber spinning has been used in fiber manufacturing since the early 1990s, and has proven to be an effective technique to reduce fiber PMD.

We will use fiber spinning technology to illustrate how PMD_Q is improved with different PMD reduction levels. We consider that a fiber PMD distribution without fiber spinning has a Maxwellian probability density function (PDF) [25]:

$$p(\tau) = \sqrt{\frac{2}{\pi}}\frac{\tau^2}{\sigma^3}\exp\left(-\frac{\tau^2}{2\sigma^2}\right) \qquad (4)$$

where τ is the PMD and σ is a coefficient that determines the fiber PMD distribution. For a given PMD distribution, one can calculate the linear average and root mean square (RMS) average of PMD:

$$\langle\tau\rangle = \frac{4}{\sqrt{2\pi}}\sigma \qquad (5)$$

$$\sqrt{\langle\tau^2\rangle} = \sqrt{3}\sigma \qquad (6)$$

From Equations (5) and (6), the relationship between the linear average and RMS average of PMD can be determined as

$$\sqrt{\langle\tau^2\rangle} = 1.085\langle\tau\rangle \qquad (7)$$

We have found that after fiber spinning, the PMD distribution remains Maxwellian with σ corrected by the PMD reduction factor. If the unspun fiber PMD is described by a distribution with σ_0, then the spun fiber PMD distribution can be determined by [25]

$$\sigma_{sp} = \xi\sigma_0 \qquad (8)$$

where ξ is the PMD reduction factor (PMDRF) that is defined by the ratio of the PMD of spun fiber to the PMD of unspun fiber. To calculate the effect of fiber spinning on fiber link PMD or PMD_Q with respect to the ITU-T reference parameters (M=20, Q=0.01%), we take five PMD distributions shown in Figure 3 with different PMDRF values ranging from 0.2 to 1. For reference, a PMDRF of 1 corresponds to the PMD distribution of fiber without spinning, while a PMDRF of 0.2 indicates that the PMD is reduced by a factor of 5 due to fiber spinning. Figure 3 shows that fiber spinning reduces the average PMD value and makes the PMD distribution narrower. These two effects contribute to a lower PMD_Q value.

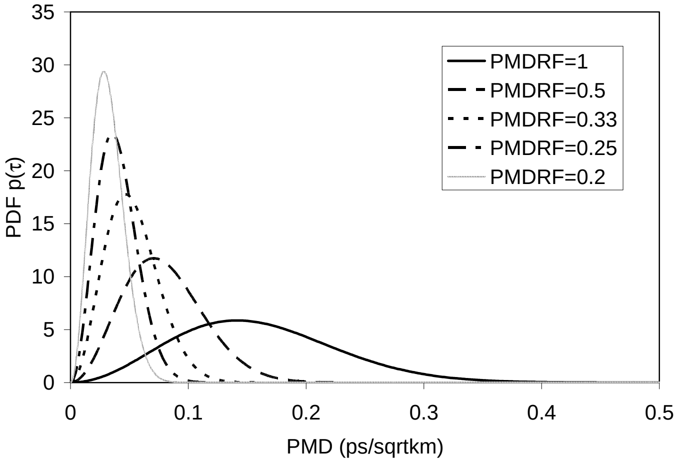

Fig. 3. PMD probability density functions for five different PMD reduction factors (PMDRF).

For each of the distributions in Figure 3, we calculated the corresponding PMD_Q value and show the results in Figure 4 as a function of 1/PMDRF, which describes the fiber spinning effectiveness. This clearly shows that the PMD_Q is reduced through fiber spinning. However, the relationship between PMD_Q and the magnitude of PMD reduction by spinning is not linear. Instead, the PMD_Q value decreases very rapidly with initial spin reduction and then becomes less responsive to higher PMD spinning effectiveness values. Figure 4 demonstrates that even a small amount of PMD reduction by fiber spinning is effective in reducing the PMD_Q value of high PMD fibers.

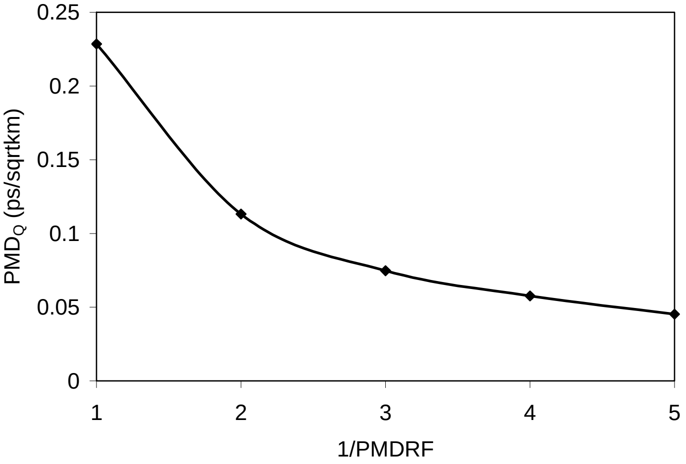

Fig. 4. The PMD_Q values resulting from the distributions presented in Figure 3 as a function of the fiber spinning effectiveness.

Another important aspect in controlling PMD_Q is to reduce PMD outliers, as even a small quantity of such outliers can significantly affect the PMD_Q value. To illustrate the effects of outliers on PMD_Q, we consider a PMD distribution composed of two Maxwellian distributions:

$$p(\tau) = (1-g)\sqrt{\frac{2}{\pi}}\frac{\tau^2}{\sigma_1^3}\exp\left(-\frac{\tau^2}{2\sigma_1^2}\right) + g\sqrt{\frac{2}{\pi}}\frac{\tau^2}{\sigma_2^3}\exp\left(-\frac{\tau^2}{2\sigma_2^2}\right) \quad (9)$$

where the subscripts 1 and 2 denote the main distribution and the outlier distribution, respectively, and g is the outlier weighting factor (WF). First, we consider outlier distributions with a fixed WF of 0.1, but different sigma values ranging from 0.02 to 0.1 ps/√km. In all cases the main distribution has a sigma of 0.02 ps/√km. When both the main and outlier distributions have a sigma of 0.02ps/√km, it means that there are no outliers in the distribution. The combined PMD distributions are plotted in Figure 5a. While the overall distributions look quite similar, the expanded scale in Figure 5b illustrates the differences in the tail regions of the distributions.

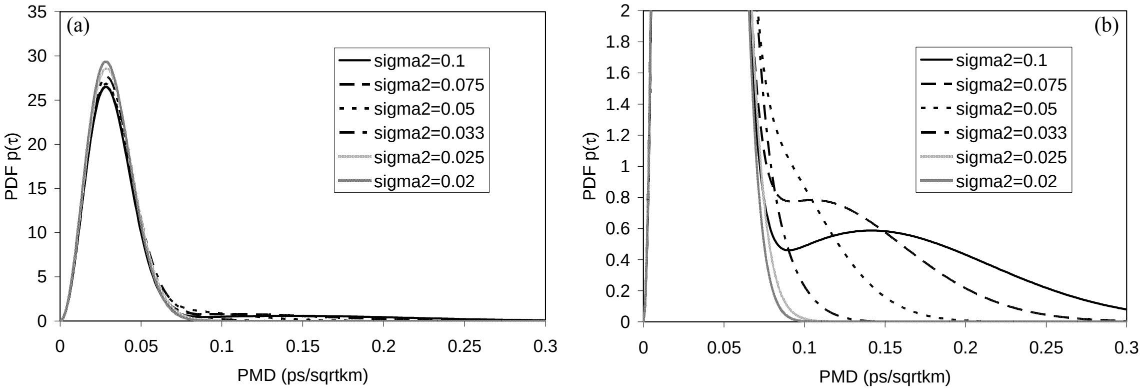

Fig. 5. (a) The composite probability density functions of PMD distributions with outlier distributions of varying sigma (b) an expanded view of the tail (outlier) regions of the distributions.

Figure 6 shows the calculated PMD_Q value as a function of the PMD outlier sigma, σ_2. The PMD outlier sigma has a significant effect on PMD_Q. For example, when the PMD outlier sigma increases from 0.02 to 0.075, the PMD_Q value is nearly doubled.

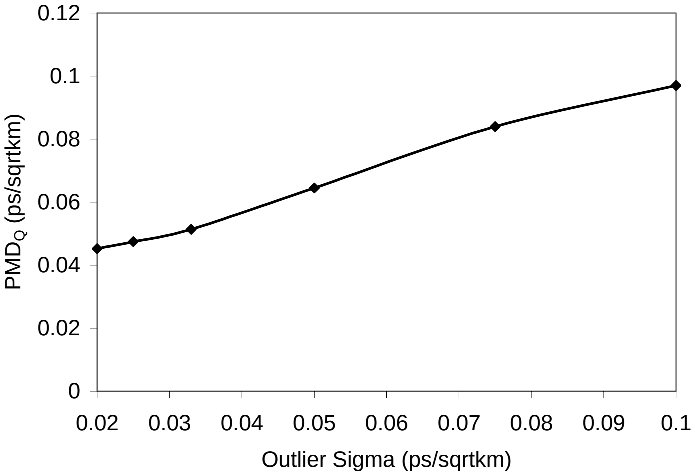

Fig. 6. The PMD_Q values resulting from the distributions presented in Figure 5 as a function of the outlier distribution sigma.

The weighting factor in Equation (9) also plays an important role in the calculation of the PMD_Q value. To demonstrate this effect we took PMD distributions with fixed σ_1 and σ_2 of 0.02 and 0.05 ps/√km, respectively, and varied the weighting factors from 0.1 to 0.5 as shown in Figure 7. The calculated PMD_Q values are plotted in Figure 8, which indicates that PMD_Q increases with increasing outlier weighting factor. From Figures 6 and 8, we conclude that PMD outliers have the potential to increase PMD_Q values significantly.

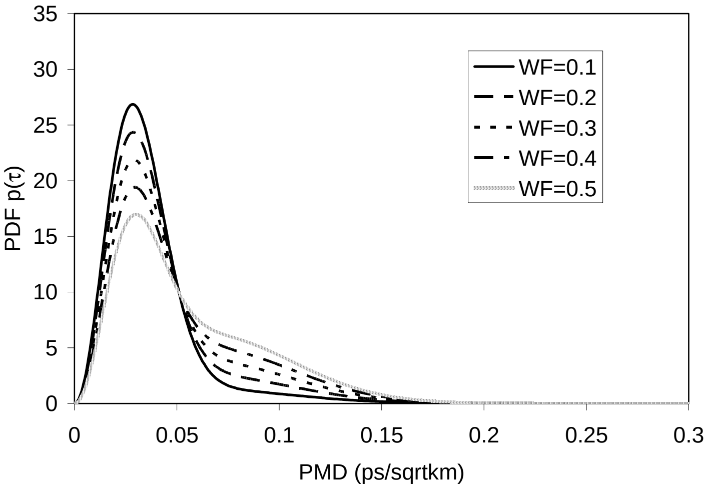

Fig. 7. Probability density functions for five different outlier weighting factors with fixed σ_1 and σ_2 of 0.02 and 0.05 ps/√km, respectively.

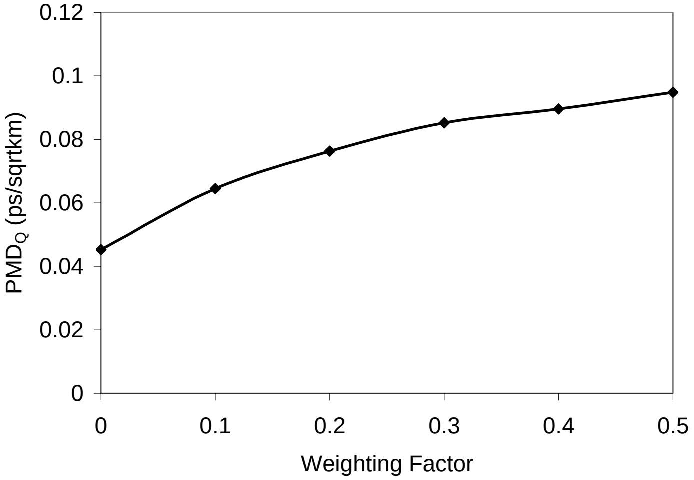

Fig. 8. The PMD$_Q$ values resulting from the distributions presented in Figure 7 as a function of outlier weighting factor.

In addition to the modeling results presented above, a practical example can be established by revisiting the distribution of cabled fiber PMD coefficients shown in Figure 1. If a handful of outliers is added to this distribution (with 3 fibers at 0.16 ps/√km and 2 fibers at 0.19 ps/√km), the calculated PMD$_Q$ value increases by approximately 8% to a value of 0.063 ps/√km. Furthermore, the probability of exceeding the end-to-end DGD target of 7.5 ps for cabled fiber increases to 7.1×10^{-8}, slightly exceeding the requirement of 6.5×10^{-8}. This example clearly shows the importance of controlling such outliers, which can be achieved through reducing asymmetry in the refractive index and stress profiles of the fiber and through introducing controlled polarization mode coupling via fiber spinning.

Other measurement techniques, such as polarization optical time domain reflectometry (POTDR), have been studied to monitor PMD uniformity and to identify PMD outliers within optical fibers or cables [26]. However, allowing manufacturers to clearly identify outliers with PMD magnitudes below certain values remains a key challenge.

6. Conclusion

This paper discusses the evolution, utilization and control of statistical PMD specifications through theory, modeling and example. Integrated into IEC and ITU-T standards, these statistical methods provide a practical and reliable means of assessing specification compliance and network system design.

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